My plenary talk:

THE ALMOST LACK OF MEMORY (ALM) DISTRIBUTIONS

AND THEIR APPLICATIONS

Abstract

Long ago baron **Augustin-Louis Cauchy (1789 – 1857)** proved in 1821that if the functional equation *f(x+y)= f(x).f(y)* holds for any non-negative arguments *x* and *y*, then the function f(x) exponential $f(x)=e^{cx}$ function. When applied to the probability property of a random lifetime of a technical unit *X* it looks P{X ≥ x+y}= P {X ≥ x}.P{X ≥ y} From Kaushy theorem and Probability it follows that the lifetime probability distribution function has the *form F* {X ≥ x}= $1 – e^{-ax}$. And it follows that if this unit still works (is alive at age y), the chances to stay alive some more time *x*, is the same as when just starts functioning:

*P{*X≥ x+y I X ≥ y }= P{X ≥ x}.

A conventional reading of this property is known as Lack-of-Memory (LM) property at any age *y*. It means that at any age *y* the units with exponentially distributed lifetimes lose the memory about their current age and behave as a just newborn. This is a characteristic property that helps in practice to recognize the lifetime distribution of technical items.

In a series of works with numerous colleagues (please, see the references) on similar properties that may be used in practice to recognize the lifetime distribution of technical items. And we found that if a lifetime shows the lack of memory at a given age *c*

It will lose the memory at any age *mc* integer multiple to the constant *c* for infinitely many times *m=2,3, 4,…* For this reason, we named these distributions ALM distributions. And we found the mathematical form of this class of probability distributions, established numerous mathematical presentations, physical properties, and found various practical applications.

My talk is focused on some published articles of these results about ALM distributions.

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