Recent results in performance modelling of finite-source retrial queues with collisions and their applications

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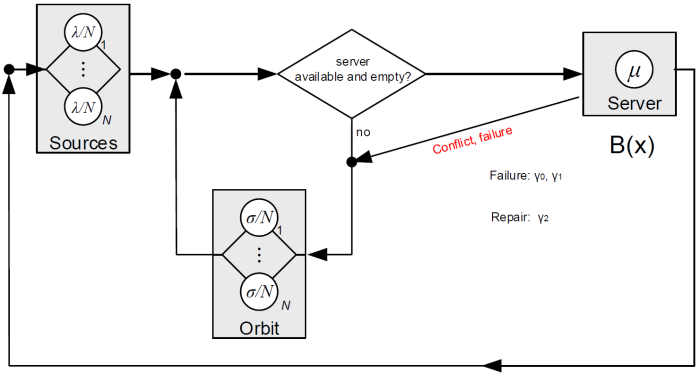
Outline

Finite source retrial queueing system with collisions



Performance measures

- *Distribution of number of requests in the system, including in service and in orbit*
- *Distribution of number of retrials*
- *Distribution of the response/waiting time of a customer*

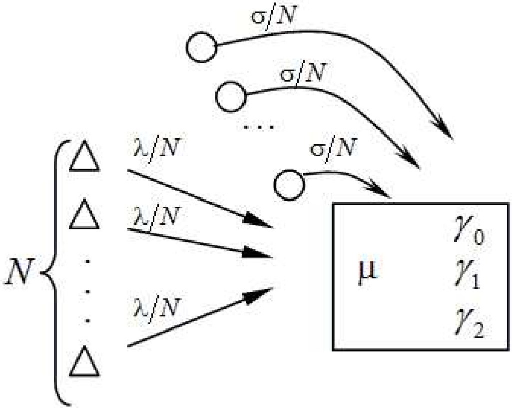
Tool supported and algorithmic approaches

- *MOSEL (Modeling, Specification and Evaluation Language) solution*
- *Algorithmic method*

Simulation approach

- *The effect of distributions of the involved random variables on the distribution of the number of customers in the system*
- *The effect of distributions of the involved random variables on the mean and variance of the response/waiting time of a request*
- *The effect of distributions of the involved random variables on the mean and variance of the number of retrials*

Asymptotic method



Asymptotic of the first order

Let $i(t)$ be number of customers in a closed retrial queueing system $M/M/1//N$ with the collisions of customers and unreliable server, then

$$\lim_{N\to\infty} E\exp\left\{jw\frac{i(t)}{N}\right\} = \exp\left\{jw\kappa_1\right\}, \tag{1}$$

where value of parameter κ_1 is the positive solution of the equation

$$\left(1-\kappa_1\right)\lambda - \mu R_1(\kappa_1) = 0, \tag{2}$$

where the stationary distributions of probabilities $R_k(\kappa_1)$ of the service state k are obtained as follows

$$R_0(\kappa_1) = \left\{ \frac{\gamma_0 + \gamma_2}{\gamma_2} + \frac{\gamma_1 + \gamma_2}{\gamma_2} \cdot \frac{a\left(\kappa_1\right)}{a\left(\kappa_1\right) + \gamma_1 + \mu} \right\}^{-1},$$

$$R_1(\kappa_1) = \frac{a\left(\kappa_1\right)}{a\left(\kappa_1\right) + \gamma_1 + \mu} \cdot R_0(\kappa_1), \tag{3}$$

$$R_2(\kappa_1) = \frac{1}{\gamma_2}\left[\gamma_0 R_0(\kappa_1) + \gamma_1 R_1(\kappa_1)\right],$$

here $a\left(\kappa_1\right)$ is

$$a\left(\kappa_1\right) = \left(1 - \kappa_1\right)\lambda + \sigma\kappa_1. \tag{4}$$

Asymptotic of the second order

$$\lim_{N\to\infty} E\exp\left\{jw\frac{i(t)-\kappa_1 N}{\sqrt{N}}\right\} = \exp\left\{\frac{(jw)^2}{2}\kappa_2\right\}, \qquad (5)$$

where the value of parameter κ_2 is defined by expression

$$\kappa_2 = \frac{\gamma_2\mu(R_1-b_1)+(1-\kappa_1)\lambda\left\{(\gamma_1+\gamma_2)\,b_1+(1-\kappa_1)\,\lambda R_2\right\}}{(\lambda+\mu b_2)\,\gamma_2-(1-\kappa_1)\,\lambda\,(\gamma_1+\gamma_2)\,b_2}, \qquad (6)$$

where

$$b_1 = \frac{(1-\kappa_1)\,\lambda}{a+\gamma_1+\mu}R_0, \qquad b_2 = \frac{(\sigma-\lambda)(R_0-R_1)}{a+\gamma_1+\mu}. \qquad (7)$$

From the proved theorem it follows that if $N \to \infty$ the limiting distributions for the centered and normalized number of customers in the system has a Gaussian distribution with variance κ_2, defined by the expression (6).

Corollary

As a consequence the distribution of the number of customers in the system is Gaussian with mean $N\kappa_1$ and variance $N\kappa_2$, respectively.

$$\lim_{N\to\infty} \mathsf{E}\exp\left\{jw\frac{T}{N}\right\} = q + (1-q)\frac{\sigma q}{\sigma q - jw}, \tag{8}$$

where value of parameter q is defined by expression

$$q = \frac{(1-\kappa_1)\lambda}{(1-\kappa_1)\lambda + \sigma\kappa_1}. \tag{9}$$

Corollary

Characteristic function of the sojourn time of the customer in the system in a prelimiting situation of finite N can be approximated by a function of the form

$$\mathsf{E}\, e^{juT} = q + (1-q)\frac{\sigma q}{\sigma q - juN}, \tag{10}$$

Let ν be the number of transitions of the tagged customer into the orbit, then

$$\lim_{N\to\infty} \mathsf{E}\, z^{\nu} = \frac{q}{1-(1-q)z}, \tag{11}$$

where value of parameter q is

$$q = \frac{(1-\kappa_1)\lambda}{a}. \tag{12}$$

Corollary

The probability distribution $P\{\nu = n\},\ n = \overline{0,\infty}$ of the number of transitions of the tagged customer into the orbit is geometric and has the form

$$P\{\nu = n\} = q(1-q)^n, \quad n = \overline{0,\infty}. \tag{13}$$

Comparisons

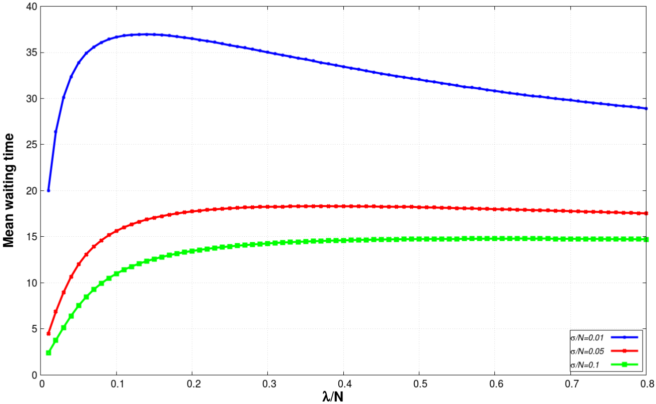


Figure: Mean waiting time in the orbit without collisions, $N = 10$

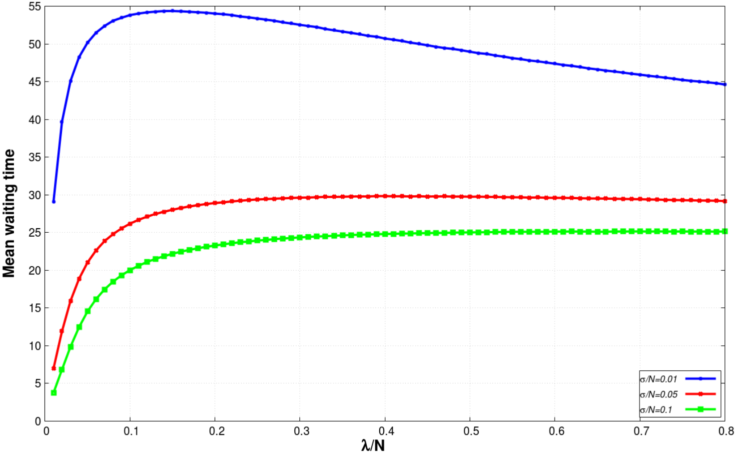


Figure: Mean waiting time in the orbit with collisions, $N = 10$

$$\lambda = 0.5, \quad \mu = 1, \quad \sigma = 5, \quad \gamma_0 = 0.1, \quad \gamma_1 = 0.2, \quad \gamma_2 = 1.$$

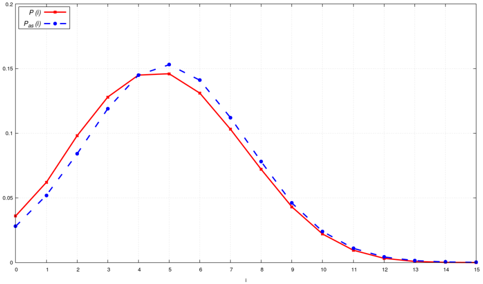


Figure: Comparison of the asymptotic and numerical results in the case $N = 15$

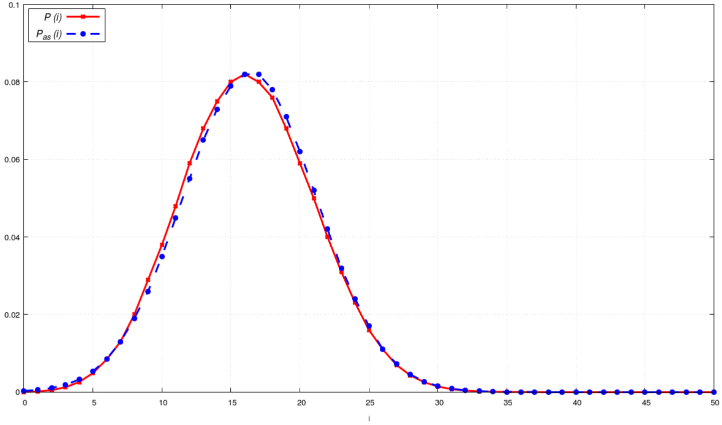


Figure: Comparison of the asymptotic and numerical results in the case $N = 50$

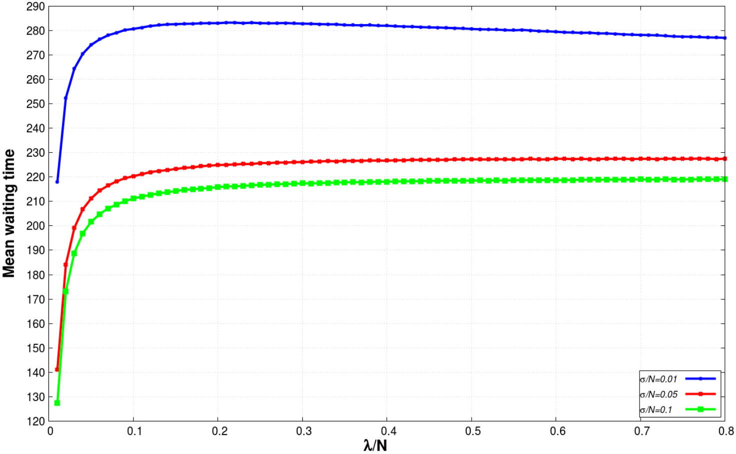


Figure: Asymptotic mean waiting time in the orbit

Kolmogorov distance Δ

$$\Delta = \max_{0 \le i < \infty} \left| \sum_{n=0}^{i} \left(P_{as}(\nu = n) - P_{s}(\nu = n) \right) \right| .$$

Realizing the simulation program for

$$\lambda = 1, \quad \mu = 1, \quad \sigma = 4, \quad \gamma_2 = 1$$

and applying the approximation (13), we will provide the Kolmogorov distance Δ for various values N and $\gamma = \gamma_0 = \gamma_1$ in the Table 1.

Table: Kolmogorov distance between distribution $P_s(i)$ and approximation of the geometric distribution $P_{as}(i)$ for various values of the parameters N and γ

	$N = 20$	$N = 30$	$N = 50$	$N = 100$	$N = 200$
$\gamma = 0.05$	0.026	0.016	0.009	0.005	0.003
$\gamma = 0.1$	0.024	0.015	0.009	0.004	0.002
$\gamma = 0.5$	0.017	0.011	0.006	0.004	0.001

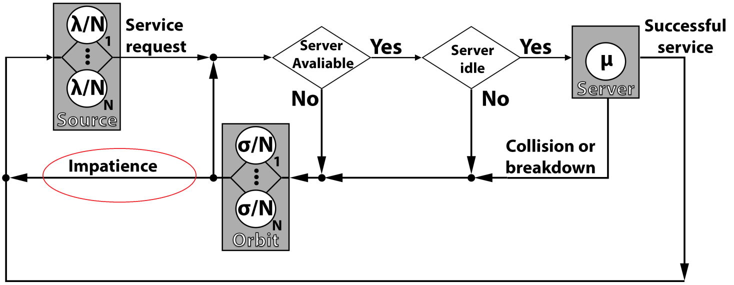


Figure: Impatient customers

Conclusions

1. Finite source retrial queueing system with collisions
2. Different solution approaches
3. Recent results on non-reliable servers using asymptotic methods
4. Graphical illustrations, comparisons

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Thank You for Your Attention